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HEAT TRANSFER TO NEWTONIAN AND NON-NEWTONIAN FLUIDS FLOWING IN A LAMINAR REGIME

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Abstract—A new analytical approach is presented to estimate the local and mean heat flux from a wall maintained at a given temperature to a fluid flowing in a laminar regime. The procedure can be applied to the analysis of plane as well as cylindrical geometry configurations while the fluid behaviour can be considered Newtonian or non-Newtonian provided the axial velocity distribution can be represented by an analytical function of position. The method is simple and gives very accurate results when compared with numerical estimates previously presented in the literature. A number of comparisons are presented in this work showing that maximum deviations are always below 3.5% in terms of mixing cup temperature. The technique is so simple that it should be useful for the analysis of similar problems in the field of convective heat and mass transfer.

NOMENCLATURE

$Ai(\ )$	Airy function
C_1	parameter defined by equation (14)
C_2	parameter defined by equation (15)
C_3	parameter defined by equation (16)
C_4	parameter defined by equation (17)
C_p	specific heat capacity
F_0	first term in equation (20)
F_1	first perturbation term in equation (20)
g	parameter defined by equation (27a)
Gz	Graetz number
k	thermal conductivity
M	constant to denote flat (0) or cylindrical geometry (1)
m	coefficient in equation (10)
N	constant index in power-law model
n	coefficient in equation (10)
$P[;]$	incomplete gamma function [see equation (35)]
p	parameter defined by equation (27b)
Pr	Prandtl number
q	parameter in equation (35)
R	tube radius or half thickness of a flat duct
r	normal coordinate measured from the axial axis of the duct
Re	Reynolds number
S	Laplace transform variable
T	dimensionless temperature
T_m	dimensionless mixing cup temperature
t	dimensional temperature
t_0	inlet fluid temperature
t_s	wall temperature
$\langle V \rangle$	average velocity in the duct
w	coefficient in equation (10)
X	transformed variable of Y when $S \to \infty$
x	axial flow coordinate
Y	dimensionless radial (pipe) or cross-channel (flat duct) coordinate
Z	dimensionless axial coordinate

Greek symbols

β	parameter in equation (28)
$\Gamma(\ )$	gamma function
γ	parameter in equation (28)
δ	parameter in equation (31)
ρ	density
τ	variable in equation (35)
ϕ	Laplace transform of T
ϕ_0	first term in equation (7), $= 1$
ϕ_1	first perturbation term in equation (7)
ϕ_2	second perturbation term in equation (7)
ϕ_3	third perturbation term in equation (7)
ω	constant in equation (35)

Superscripts

$'$	denotes first derivative
$''$	denotes second order derivative

1. INTRODUCTION

IN MANY industrial applications, heating or cooling is achieved by allowing the liquid to flow in ducts, or in the form of a thin film, where the solid boundaries are maintained at a different temperature from that of the bulk. There are many practical situations in which the velocity field is fully developed but not the temperature field. For example a molten polymer, because of its high viscosity, can be regarded as a fully developed flow and due to its low thermal conductivity will present an undeveloped temperature field. Other examples can be found in food engineering processes related with the cooling and heating of fluid streams. Moreover the rheological behaviour of many of these fluids can only be described with a non-Newtonian model.

Possibly due to these reasons, investigations in this

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field have led to a large body of literature, including entire books. When the flow is steady and laminar and the fluid properties are assumed to be constant, the so called Graetz–Nusselt problem is met. Its solution allows the prediction of the mixed-cup temperature along the system and the local flux at the heating or cooling surface. This is not, however, very simply solved.

Since the problem is linear the analytical solution can be obtained by the method of separation of variables and takes the form of an infinite sum of eigenfunctions. The series does not converge very rapidly when the duct is short. This difficulty is avoided by using a sort of Lévêque [1] solution for small contact times and the series solution for large contact times.

The case of a Newtonian fluid was repeatedly solved with different analytical approaches by many authors after the pioneering works of Graetz [2] and Nusselt [3], until Brown [4] presented his accurate work in which up to eleven eigenvalues were determined by a very precise numerical technique. Brown's [4] numerical results can be considered as the most accurate for the case of Newtonian fluids. They can be used in circular pipes or in flat ducts.

In the case of non-Newtonian fluids the situation is worse as the eigenfunctions must be determined, with considerable effort, for each particular case, Lyche and Bird [5] investigated the problem of non-Newtonian fluids with a 'power law' model which has been proven useful for estimating the pressure drop in flow systems. It is interesting to mention that they only presented four eigenvalues for integer values of the inverse power index of the non-Newtonian model. More recently Suckow *et al.* [6] presented a contribution which provides the first two eigenvalues for a dilatant non-Newtonian fluid in which the inverse power index was 0.5. When the method is compared with previous results for Newtonian fluids a rather poor agreement is found due to the small number of calculated eigenvalues.

On the other hand, some attempts have been made to extend Lévêque's [1] solution, such as those of Shih and Tsou [7] and Richardson [8]. However in spite of these great efforts, the range of validity for the expressions obtained is not known but they are certainly valid for short tube length or large Graetz (Gz) numbers.

The intention of this contribution is to develop a rapid analytical procedure to estimate the local heat flux and the mixing-cup temperature along the duct which can be applied to any kind of non-Newtonian fluid model and geometrical flow configuration with a certain degree of symmetry. Also, the method can produce results valid in the whole range of Graetz numbers ($0 \leqslant Gz \leqslant \infty$).

To achieve these purposes a matching technique will be used. Asymptotic expressions of local flux deduced in the Laplace transform field, valid for small and large values, are matched with a suitable expression. When antitransformed it provides an analytical expression for the local flux. Finally the results obtained are compared with the corresponding results generated by the infinite series referred to above. The agreement is excellent, as will be shown below.

2. ANALYSIS

Assuming laminar flow, constant physicochemical properties of the fluid, and a power law model for describing the rheological behaviour of the fluid, the dimensionless energy balance can be written as

$$\frac{\partial}{\partial Y}\left(Y^M \frac{\partial T}{\partial Y}\right) = Y^M(1-Y^{N+1})\frac{\partial T}{\partial Z} \tag{1}$$

where

$$T = \frac{t-t_0}{t_s-t_0}, \quad Y = \left(\frac{r}{R}\right), \tag{2a,b}$$

$$Z = n(M+1)(x/R)/(Re\,Pr), \tag{2c}$$

$$Re\,Pr = \frac{\rho C_p \langle V \rangle R}{k}, \tag{2d}$$

t being the dimensional temperature, x the axial flow coordinate, k the thermal conductivity of the fluid, $\langle V \rangle$ the average velocity in the duct, R the tube radius or half thickness of a flat duct, r the normal coordinate measured from the axial axis of the duct, C_p the specific heat capacity, ρ the fluid density and $M = [0, 1]$ is to denote flat and cylindrical geometry. Parameter n will be defined below.

In writing equation (1) it is assumed that molecular axial transport and mechanical energy dissipation are negligible. Since the problem is linear, equation (1) will be solved subject to the following initial and boundary conditions:

$$T = 0, \quad Z = 0 \quad 1 > Y \geqslant 0, \tag{3a}$$

$$T = 1, \quad Z \geqslant 0 \quad Y = 1, \tag{3b}$$

$$\frac{\partial T}{\partial Y} = 0, \quad Z \geqslant 0 \quad Y = 0. \tag{3c}$$

Solutions to other situations can be found by a suitable application of the superposition principle. By defining

$$\phi = S \int_0^\infty T \exp(-SZ)\,\mathrm{d}Z. \tag{4}$$

Equation (1) can be reduced to the following ordinary differential equation:

$$\frac{\mathrm{d}}{\mathrm{d}Y}\left(Y^M \frac{\mathrm{d}\phi}{\mathrm{d}Y}\right) = Y^M(1-Y^{N+1})S\phi \tag{5}$$

with the boundary conditions

$$\phi = 1, \quad Y = 1, \qquad \phi' = 0, \quad Y = 0 \tag{6a,b}$$

where the prime denotes first derivative with respect to Y.

A general solution to equation (5) would lead to a series expression which should be avoided. Rather asymptotic solutions for small and large S values will be attempted. In fact when $S \to 0$, equation (5) itself suggests the following series as a solution:

$$\phi = \phi_0 + S\phi_1 + S^2\phi_2 + S^3\phi_3 + O(S^4) \quad (7)$$

which once replaced into equation (5) and terms of like power of S are equated generates the following set of independent ordinary differential equations:

$$\frac{d}{dY}\left(Y^M \frac{d\phi_i}{dY}\right) = Y^M(1-Y^{N+1})\phi_{i-1} \quad (8)$$

with $i = 1, 2, \ldots$ and $\phi_0 = 1$. Clearly equations (8) must be solved with

$$\phi_i(1) = 0; \quad \phi_i'(0) = 0. \quad (9a,b)$$

After some simple algebraic manipulations it can be shown that

$$\left.\frac{1}{S}\frac{d\phi}{dY}\right|_{Y=1} = n + mS + wS^2 + O(S^3) \quad (10)$$

where

$$n = \phi_1'(1) = (M+1)^{-1} - (N+M+2)^{-1}, \quad (11)$$

$$m = \phi_2'(1) = 2[(M+1)(M+3)]^{-1} - [C_1 + (M+1)^{-1}](N+M+4)^{-1} + [(N+M+2)(N+3)(2N+M+5)]^{-1} + C_1 n, \quad (12)$$

$$w = \phi_3'(1) = [8(M+1)(M+3)(M+5)]^{-1} - \{C_3 + [8(M+1)(M+3)]^{-1}\} \times (N+M+6)^{-1} + (C_3+C_4)(2N+M+7)^{-1} - C_4(3N+M+8)^{-1} + C_1 m + (C_2 - C_1^2)n \quad (13)$$

with

$$C_1 = [(N+M+2)(N+3)]^{-1} - [2(M+1)]^{-1}, \quad (14)$$

$$C_2 = C_3 - C_4 + C_1^2 - [8(M+1)(M+3)]^{-1}, \quad (15)$$

$$C_3 = [C_1 + (M+1)^{-1}]/[(N+M+4)(N+5)], \quad (16)$$

$$(1/C_4) = 2(N+M+2)(N+3)^2(2N+M+5). \quad (17)$$

On the other hand when $S \to \infty$, a change of the independent variable Y is suggested,

$$X = S^{1/3}(1-Y). \quad (18)$$

By replacing Y by X in equation (5) and expanding by the binomial theorem the resulting expressions yield

$$\frac{d^2\phi}{dX^2} = MS^{-1/3}\frac{d\phi}{dX} + (N+1)X\phi \times [1 - (NX/2)S^{-1/3}] + O(S^{-2/3}). \quad (19)$$

Equation (19) can be solved by the following series solution:

$$\phi = F_0(X) + S^{-1/3}F_1(X) + O(S^{-2/3}). \quad (20)$$

By introducing equation (20) into equation (19) and by collecting terms of equal power of S, the following system of ordinary differential equations results:

$$F_0'' = (N+1)XF_0, \quad (21)$$

$$F_1'' - MF_0' = (N+1)XF_1 - (N+1)(N/2)X^2F_0 \quad (22)$$

subject to

$$F_0(0) = 1, \quad F_1(0) = F_0(\infty) = F_1(\infty) = 0 \quad (23)$$

where primes and double primes denote first and second derivatives with respect to X. The general solution to equation (21) is well known in terms of Airy functions [$A_i(\ )$] [9]

$$F_0 = A_i[(N+1)^{1/3}X]/A_i(0) \quad (24)$$

while equation (22) only needs a particular solution, which can be written in terms of F_0,

$$F_1 = (0.5M + 0.1N)XF_0 - 0.1NX^2F_0'. \quad (25)$$

Thus when $S \to \infty$, the following asymptotic expression is found for the flux in the transformed field:

$$\left.\frac{1}{S}\frac{d\phi}{dY}\right|_{Y=1} = gS^{-2/3} - p/S \quad (26)$$

where

$$g = -A_i'(0)(N+1)^{1/3}/A_i(0), \quad (27a)$$

$$p = 0.5M + 0.1N. \quad (27b)$$

Expressions (10) and (26), valid for small and large values of S respectively, should be matched by a suitable rational expression in such a way that its expansions, for small and large values of S, should coincide with equations (10) and (26), respectively. The first proposal is

$$\left.\frac{1}{S}\frac{d\phi}{dY}\right|_{Y=1} = \frac{g(S+\beta)^{1/3}}{(S+\gamma)} \quad (28)$$

provided the unknowns β and γ fulfil the following equations:

$$g\beta^{1/3} = n\gamma, \quad (29)$$

$$(1/3)g\beta^{-2/3}\gamma^{-1} - g\beta^{1/3}\gamma^{-2} = m. \quad (30)$$

Thus equation (28) will coincide with equation (10) up to terms of order S and with equation (26) up to terms of order $S^{-2/3}$. However equation (28) can be anti-transformed if $\beta > \gamma$, as will be shown below. In those cases where such a condition is not met a more accurate rational expression is needed. A new trial is made with

$$\left.\frac{1}{S}\frac{d\phi}{dY}\right|_{Y=1} = \frac{g(S+\beta)^{1/3}}{(S+\gamma)} - \frac{pS}{(S+\delta)^2}. \quad (31)$$

By expanding equation (31) for small and large values of S and comparing with equations (10) and (26), respectively, the three unknowns must fulfil the following system of algebraic equations:

$$3mn^2\gamma^3\delta^2 = -3n^3\delta^2\gamma^2 + g^3\delta^2 - 3n^2p\gamma^3, \quad (32)$$

$$9n^6\gamma^4\delta^3 - 3n^3g^3\gamma^2\delta^3 - g^6\delta^3 + 18pn^5\gamma^6 - 9wn^5\gamma^6\delta^3 = 0. \quad (33)$$

After solving the system (32)–(33), β can be calculated from equation (29). It should be noticed that equation (31) coincides exactly with equations (10) and (26) for small and large values of S respectively although the algebraic complexity of the problem has increased. In any case it is not difficult to reduce the system (32)–(33) in a unique non-linear algebraic expression with only one unknown which can be solved by Newton–Raphson methods. It should be stressed that equation (28) is a particular case of equation (31). Thus by taking the antitransform of equation (31) the expression for local flux is found,

$$\left.\frac{\partial T}{\partial Y}\right|_{Y=1} = [gZ^{-1/3}\exp(-\beta Z)/\Gamma(2/3)] + g(\beta-\gamma)^{1/3}\exp(-\gamma Z)P[\tfrac{2}{3};(\beta-\gamma)Z] - p\frac{\mathrm{d}}{\mathrm{d}Z}[Z\exp(-\delta Z)] \quad (34)$$

where $\Gamma(\ )$ denotes gamma function and $P[;]$ the incomplete gamma function:

$$P[q;\omega] = \frac{1}{\Gamma(q)}\int_0^{\omega}\tau^{q-1}\exp(-\tau)\,\mathrm{d}\tau. \quad (35)$$

The mixing-cup temperature can be calculated from equation (34) taking into account its definition and the energy balance [equation (1)]:

$$T_m = \frac{1}{n}\int_0^1 Y^M(1-Y^{N+1})T\,\mathrm{d}Y = \frac{1}{n}\int_0^Z \left.\frac{\partial T}{\partial Y}\right|_{Y=1}\mathrm{d}Z. \quad (36)$$

Introducing equation (34) into equation (36) yields

$$T_m = P[\tfrac{2}{3};\beta Z] - \left(\frac{\beta-\gamma}{\beta}\right)^{1/3}\exp(-\gamma Z) \times P[\tfrac{2}{3};(\beta-\gamma)Z] - (p/n)Z\exp(-\delta Z). \quad (37)$$

Expression (37) should be useful in the range of all Z values ($0 \leqslant Z \leqslant \infty$) representing a continuous function of Z for each particular situation (M and N given). It should be stressed that the procedure can be applied to any other kind of rheological model provided the velocity distribution function is known. Moreover the case of shear stress dependent thermal diffusivity could be handled with exactly the same procedure.

It is possible to find an expression for the local flux and mixing-cup temperature by applying the superposition or convolution theorem when other boundary conditions must be considered.

3. RESULTS AND DISCUSSION

Though most previous works are based on a cylindrical geometric configuration, it is also interesting to analyze the accuracy of approximate results obtained in flat ducts. Figure 1 presents deviations between approximate values of T_m calculated with equation (37) and those generated by the very accurate numerical calculations of Brown [4] for Newtonian fluids ($N = 1$). For plane geometry ($M = 0$) there is no need to take into account the second term of equation (31) which is equivalent to assuming $p = 0$ in equation (37). Under these conditions the method is straightforward since the only difficulty is the numerical calculation of the incomplete gamma function $P[;]$ which can be performed by the series suggested by Zelen [10]. It is shown that the maximum deviation is about 3.5% in the whole range of Z values. Results of Suckow *et al.* [6] are also presented for comparison purposes ($N = 1$) showing that their results can only be considered accurate when $Z > 0.5$. This conclusion should have been expected since only two eigenfunctions were evaluated by Suckow *et al.* [6]. However since their results are the only available heat transfer calculations for non-Newtonian fluids in flat ducts these and our approximations are presented in Table 1 ($N = 0.5$). Once again it is shown that the agreement is fair when $Z > 0.5$. Taking into account the case of Newtonian fluids, we can safely conclude that our approximate procedure will predict fairly accurate values of T_m in the whole range of Z values. When heat transfer in tubes is considered ($M = 1$) it is not possible to find a suitable root of γ with equations (29) and (30). Thus the more complex system (32)–(33) has to be solved for γ and δ. Once again the mixing-cup temperature (T_m) calculated with equation (37) is graphically compared in Fig. 2 with the very accurate results of Brown [4] for $N = 1$. The agreement is excellent, better than in the case of flat ducts since in this case ($M = 1$) more parameters are needed to fit the asymptotic equations of the local flux in the Laplace field with the rational expression. It should be noted that when $Z < 0.002$ Brown's [4] solution begins to depart from the exact solution. As pointed out in the introduction a series solution would need a con-

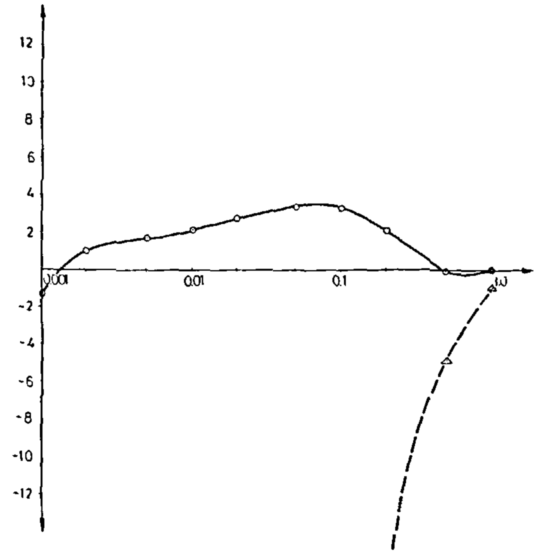


FIG. 1. Percentage deviations between our T_m estimates (full line) and Suckow *et al.*'s [6] (broken line) with almost exact Brown's [4] values. Plane geometry ($M = 0$) and Newtonian fluids ($N = 1$).

Table 1. Comparison of T_m values obtained in this work with those of Suckow *et al.* [6]. Plane geometry ($M = 0$) and $N = 0.5$

Z	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1
This work	0.015409	0.024463	0.045079	0.071597	0.113738	0.209454	0.330092	0.507401	0.798038	0.953912
Suckow *et al.* [6]	0.084093	0.081402	0.074539	0.066698	0.061739	0.096028	0.204511	0.409649	0.762305	0.947831

siderable number of terms to predict the flux for such small values of Z. In this region our solution coincides exactly with the asymptotic solution of Shih and Tsou [7] (dashed line Fig. 2) which is shown to be valid up to $Z \simeq 0.1$. Incidentally it can be shown that our simple analytical asymptotic solution deduced in this work with only two terms, once inverted, also coincides with the numerical solution of Shih and Tsou [7] up to $Z \simeq 0.1$. (The maximum deviation at $Z = 0.1$ is about 4%.)

In Table 2 our results are compared with tabulated values of Lyche and Bird [5]. The agreement is excellent within the range in which Lyche and Bird's [5] results apply ($Z \geqslant 0.05$).

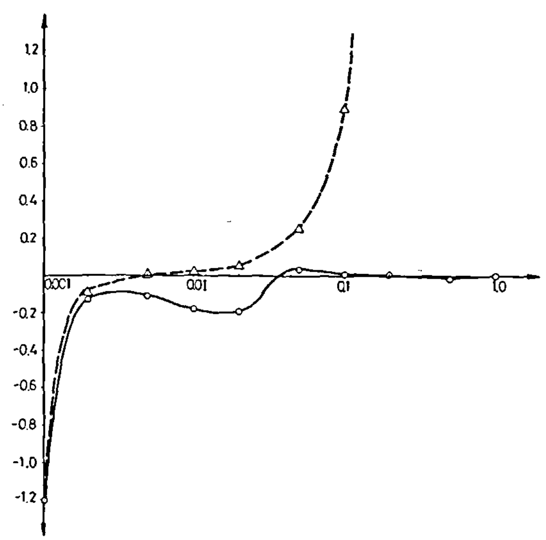
It can be concluded that the method presented here can be very useful, due to its simplicity, to estimate the flux and the mixing-cup temperature in laminar flow configurations formed by non-Newtonian fluids. It is not restricted to any particular rheological model. Once the velocity profile is known the method can be used to estimate the values of n, m, w, g and p. Furthermore the method should also be useful to estimate the rate of dissolution of a solid by the effect of a laminar flow of a non-Newtonian fluid.

It is interesting to note that the method herewith presented demands much less effort than the extended Lévêque solution of Shih and Tsou [7] or Richardson [8] since our procedure does not require the numerical solution of any ordinary differential equations. Moreover the results of Shih and Tsou [7] are only valid in a limited range of Z values as shown in Fig. 2 (dashed curve) for the case of $N = 1$ and $M = 1$. In fact when $Z > 0.2$ their results are no longer valid.

Finally our procedure can be extended to analyze other related situations such as the effect of mechanical energy dissipation or cases where boundary conditions are given in terms of flux rather than the solid–fluid temperature.

Table 2. Comparison of Brown's (almost exact) results with those obtained in this work, Shih and Tsou [7] and Lyche and Bird [5]. Cylindrical geometry ($M = 1$) and Newtonian fluids ($N = 1$)

Z	Brown [4]	This work	Shih and Tsou [7]	Lyche and Bird [5]
0.001	0.038715	0.038247	0.038251	—
0.002	0.059736	0.059659	0.059683	—
0.005	0.106572	0.106451	0.106580	—
0.01	0.163781	0.163482	0.163814	—
0.02	0.248894	0.248405	0.249035	—
0.05	0.421213	0.421350	0.422248	0.421
0.1	0.604701	0.605891	0.610085	0.605
0.2	0.810290	0.810310	0.840523	0.810
0.5	0.978856	0.978675	1.098401	0.979
1	0.999454	0.999514	0.942796	0.999



IG. 2. Percentage deviations between our T_m estimates (full ne) and Shih and Tsou's [7] (broken line) with almost exact rown's [4] values. Cylindrical geometry ($M = 1$) and Newtonian fluids ($N = 1$).

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TRANSFERT THERMIQUE A DES FLUIDES NEWTONIENS ET NON-NEWTONIENS EN ECOULEMENT LAMINAIRE

Résumé—On présente une nouvelle approche analytique pour estimer la flux thermique local ou moyen entre une paroi maintenue à une température donnée et un fluide qui s'écoule en régime laminaire. La procédure peut être appliquée à l'analyse des configurations planes aussi bien que cylindriques, tandis que le fluide peut être newtonien ou non, pourvu que la distribution de vitesse axiale soit représentée par une fonction analytique de position. La méthode est simple et elle donne des résultats très précis en comparaison des résultats numériques présentés antérieurement. Quelques comparaisons sont données dans cet article qui montre que les déviations maximales sont toujours inférieures à 3,5% en terme de température de mélange. La technique est si simple qu'elle peut être utile dans l'analyse de problèmes semblables dans le domaine du transfert convectif de chaleur et de masse.

WÄRMEÜBERTRAGUNG AN NEWTONSCHE UND NICHTNEWTONSCHE FLUIDE UNTER LAMINAREN BEDINGUNGEN

Zusammenfassung—Es wird ein neuer analytischer Ansatz zur Bestimmung des örtlichen und mittleren Wärmestroms von einer Wand, die auf konstanter Temperatur gehalten wird, an ein laminar strömendes Fluid vorgeschlagen. Das Verfahren kann sowohl auf ebene Flächen als auch Zylinder angewandt werden und sowohl bei Newtonschen als auch nicht-Newtonschen Fluiden, vorausgesetzt, daß die axiale Geschwindigkeitsverteilung als analytische Fuktion des Ortes angegeben werden kann. Das Verfahren ist einfach und liefert sehr genaue Ergebnisse, wenn man es mit numerischen Näherungsverfahren vergleicht, wie sie in der Literatur zu finden sind. In der Arbeit wird anhand mehrerer Vergleiche gezeigt, daß die maximale Abweichung immer weniger als 3,5% bezogen auf die Mischungstemperatur, ausmacht. Das Verfahren ist so einfach, daß es mit Vorteil auf die Untersuchung ähnlichter Probleme im Gebiet der konvektiven Wärme- und Stoffübertragung angewandt werden könnte.

ТЕПЛОПЕРЕНОС К НЬЮТОНОВСКИМ И НЕНЬЮТОНОВСКИМ ЖИДКОСТЯМ ПРИ ТЕЧЕНИИ В ЛАМИНАРНОМ РЕЖИМЕ

Аннотация—Представлен новый аналитический метод оценки локальной и средней величины теплового потока от стенки, поддерживаемой при заданной температуре, к потоку жидкости при ламинарном течении жидкости. Метод можно использовать для анализа как плоской, так и цилиндрической геометрии, а поведение жидкости может рассматриваться как ньютоновское или неньютоновское, если распределение осевой скорости представимо в виде аналитической функции пространственной координаты. Метод прост и дает весьма точные результаты при сравнении с имеющимися в литературе численными оценками.

Проведено несколько сравнений, показывающих, что максимальная погрешность не превышает 3,5% по температуре смешения.

Метод настолько прост, что его следует использовать для аналогичных задач конвективного тепло-и массопереноса.